Course Program
“Differential Equations”

Origin of the theory of differential equations. Differential equations in applied problems. Basic concepts and objects.

Solving the certain types of first-order differential equations. Equations with separated variables. Linear equation. The Bernoulli equation. The Riccati equation. Quasihomogeneous equations.

Scalar autonomous equation of the first order. Analysis of the model of a one-species population. Scalar linear periodic equation.

The exact differential equation. The integrating factor.

Existence and uniqueness of a solution to the Cauchy problem. Peano existence theorem. Picard theorem of existence and uniqueness. Extension of a solution to the Cauchy problem.

Elements of geometrical analysis for first order differential equations. Geometrical interpretation of the first-order differential equation. The field of directions. Integral curves. Investigation scheme of the behavior of integral curves. Kneser theorem. The comparison theorem.

Equations in symmetric form and two-dimensional autonomous systems. Vector field. Autonomous system. Investigation of the Lotka-Volterra model. Classification of phase portraits of autonomous systems in a neighborhood of the equilibrium point in the linear approximation. Node. Saddle. Bicritical node. Degenerate node. Focus. Center. On correctness of the linearization method. Grobman-Hartman theorem. On center-focus problem.

Implicit differential equation. Theorem of existence and uniqueness of a solution. Parametrization method. The Clairaut equation. The Lagrange equation. Geometry of the implicit equation. Discriminant curves and singular solutions.

Integration of higher order differential equations. Reduction of the order for special types of higher order differential equations. Equation that does not contain the required function in the explicit form. Autonomous equations. Equation that is homogeneous with respect to the sought function and its derivatives. Quasihomogeneous equation. Equations in the form of total derivative.

General theory of linear equations. Theorem of existence and uniqueness of a solution to the linear homogeneous system. Fundamental system of solutions and the general solution to a linear homogeneous higher order equation (LHE). Construction of a linear homogeneous equation by its fundamental system of solutions. The Ostrogradsky-Liouville formula. On the integration of a linear homogeneous equation in quadratures.

Linear n-th order homogeneous equations with constant coefficients. The cases of simple and multiple roots of the characteristic polynomial. Euler equation. Linear inhomogeneous equations. Method of variation of arbitrary constants (the Lagrange method). Linear inhomogeneous equations with quasipolynomial in the right-hand side. Method of indeterminated coefficients. Nonresonant and resonant cases. Method of complex amplitudes.

Linear homogeneous systems (LHS). Linear systems with constant coefficients. The Euler method. Generalization of the Euler method. Matrix exponential.

Linear inhomogeneous systems. Method of variation of arbitrary constants (the Lagrange method).Linear inhomogeneous system with constant matrix and quasipolynomial free term. Method of indeterminated cofficients.

Oscillatory behavior of solutions to linear second-order differential equations. Basic theorems of oscillatory behavior: the comparison theorem, the Sturm theorem, the nonoscillatory theorem. Existence of the infinity of zeros on the semiaxis.

Second-order linear differential equations with regular singular points. Constructing the solutions by means of generalized power series. Gauss, Legendre and Bessel equations.

Boundary-value problems. Green function.

Basic properties of solutions to systems of differential equations. Peano theorem. Uniqueness and extension of solutions. Correctness of the Cauchy problem. Stability of solutions with respect to perturbations of the initial data and the right-hand side on a finite time interval. Properties of the solution to a normal system as a function of the initial data and parameters. Continuity in the natural domain of definition. Differentiability of the solution to the Cauchy problem with respect to initial data and parameters. Asymptotic expansions of solutions to differential equations.

Theory of first integrals. Definition, geometrical interpretation, and analytic criterion of the first integral. Functionally independent first integrals. Complete collection of the first integrals. Solving the Cauchy problem by means of complete collection of the first integrals. First integrals of an autonomous system and a system in symmetric form.

Basic notions of the Lyapunov stability theory. Stability of linear systems. Stability of a linear system with constant matrix. Theorem of stability in the first approximation. Lyapunov functions. Lyapunov stability theorems. Chetaev instability theorem.

First-order partial differential equation. Linear and quasilinear first-order partial differential equations. Method of characteristics. Cauchy problem. Hopf equation.