

Course Program

“Integral Equations and Elements of Functional Analysis”

Concept of integral equation. Integral equations in applied problems. Classification of integral equations.

Fredholm integral equation of the second kind with degenerate kernel. Reduction of the problem of solvability of integral equation to the problem of solvability of the system of linear algebraic equations. Fredholm determinant. Characteristic values and functions. Conjugate equation. Fredholm theorems. Fredholm alternative.

The contraction mapping principle. Banach fixed-point theorem. Generalisation of Banach theorem. Application of the principle of contraction mappings to the solving of Fredholm integral equation of the second kind and Volterra equation of the second kind.

Application of the principle of contraction mappings to the solving of some classes of non-linear integral equations.

Linear normed spaces. Banach spaces. Linear operators. Continuity and boundedness. Norm of linear operator. Space of linear and bounded operators. Convergence in the space $L(E, E_1)$.

Product of linear continuous operators. Inverse operator. Properties of inverse operator. Banach theorem (on inverse operator).

Fredholm integral equation of the second kind. Neumann series. Iterated kernels. Resolvent. The properties of resolvent. Volterra integral equation of the second kind. Construction of the resolvent.

Fredholm theorems for the Fredholm equation of the second kind with continuous kernel. Resolvent set and spectrum of linear operator.

Compact operators. Properties. Hilbert space. Conjugate operator. Riesz-Schauder equation. Symmetric operators on the Hilbert space. Compact symmetric operators on the Hilbert space. Characteristic values and vectors of compact symmetric operator.

Orthogonalisation of the system of linearly independent elements on the linear space with scalar product.

Hilbert-Schmidt theorem. Application of the Hilbert-Schmidt theorem to the solving of operator equations. Schmidt formulas. Integral equations with symmetric kernel.

The definition of well-posed problem. Analysis of the Fredholm integral equation of the first kind in the framework of well-posed problem.

Volterra equation of the first kind. Reduction to the Volterra equation of the second kind.

Volterra integral equations with the kernel $K(t, s) = \frac{1}{(t-s)^\alpha}$, $0 < \alpha < 1$. Abel equation.

Integral-differential equations. Examples of problems which lead to integral-differential equations. Integral-differential equations with integral operators of Volterra and Fredholm types.